Intertemporal Choice Budget Constraint and Interest Rate Change

A consumer receives an income of $y_1 = 50$ in period 1 and $y_2 = 80$ in period 2. The consumer may save or borrow at an interest rate r. The intertemporal budget constraint is given by

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r},$$

where c_1 and c_2 denote consumption in periods 1 and 2, respectively.

(a) Write the consumer's intertemporal budget constraint explicitly for $r = 0.10$ and for $r = 0.20$.

(b) Compute the period-1 (horizontal) and period-2 (vertical) intercepts for both cases.

(c) Sketch both budget lines on the (c_1, c_2) plane and explain how the increase in the interest rate affects the slope and intercepts.

Solution

(a) The general intertemporal budget constraint is

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}.$$

For $r = 0.10$: Substitute $y_1 = 50$ and $y_2 = 80$:

$$c_1 + \frac{c_2}{1.10} = 50 + \frac{80}{1.10}.$$

Calculating the right-hand side,

$$\frac{80}{1.10} \approx 72.73, \quad \text{so} \quad c_1 + \frac{c_2}{1.10} \approx 50 + 72.73 = 122.73.$$

Thus, for $r = 0.10$ the constraint is

$$c_1 + \frac{c_2}{1.10} = 122.73.$$

For $r = 0.20$: Similarly, with $r = 0.20$,

$$c_1 + \frac{c_2}{1.20} = 50 + \frac{80}{1.20}.$$

Since

$$\frac{80}{1.20} \approx 66.67, \quad \text{we have} \quad c_1 + \frac{c_2}{1.20} \approx 50 + 66.67 = 116.67.$$

Thus, for $r = 0.20$ the constraint is

$$c_1 + \frac{c_2}{1.20} = 116.67.$$

(b) To find the intercepts, we rewrite the constraint in slope-intercept form. Multiply the constraint by the denominator:

For $r = 0.10$: Multiplying by 1.10:

$$1.10\, c_1 + c_2 = 1.10 \times 122.73 \approx 135.00.$$

Thus,

$$c_2 = 135.00 - 1.10\, c_1.$$

Horizontal intercept: Set $c_2 = 0$:

$$1.10\, c_1 = 135.00 \quad \Longrightarrow \quad c_1 \approx \frac{135.00}{1.10} \approx 122.73.$$

Vertical intercept: Set $c_1 = 0$:

$$c_2 = 135.00.$$

For $r = 0.20$: Multiplying by 1.20:

$$1.20\, c_1 + c_2 = 1.20 \times 116.67 = 140.00.$$

Thus,

$$c_2 = 140.00 - 1.20\, c_1.$$

Horizontal intercept: Set $c_2 = 0$:

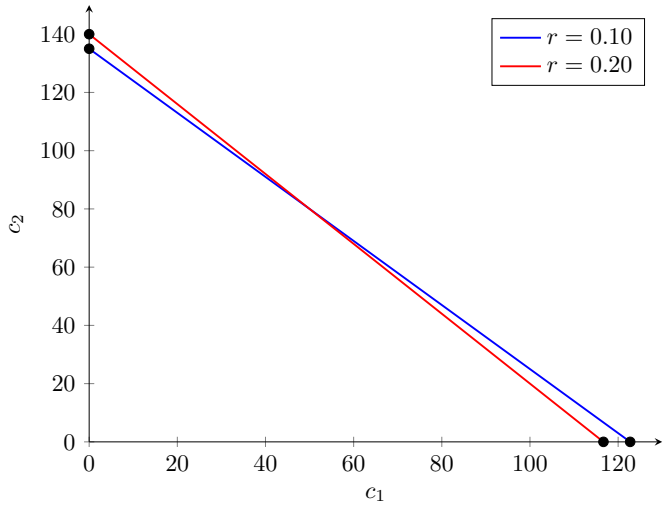
$$1.20\, c_1 = 140.00 \quad \Longrightarrow \quad c_1 \approx \frac{140.00}{1.20} \approx 116.67.$$

Vertical intercept: Set $c_1 = 0$:

$$c_2 = 140.00.$$

(c) *Graphing the Budget Lines:*

Below is a sketch of the intertemporal budget lines for $r = 0.10$ and $r = 0.20$.



Explanation: - For $r = 0.10$, the consumer's intertemporal budget line has a horizontal intercept of approximately 122.73 and a vertical intercept of 135, with a slope of -1.10. - For $r = 0.20$, the horizontal intercept decreases to approximately 116.67 (since the present value of future income declines with a higher interest rate) while the vertical intercept increases to 140 (since the future value of period 1 income is higher). The slope becomes steeper, at -1.20, indicating that the opportunity cost of consuming today (in period 1) is higher when the interest rate increases.